

# Bearing Capacity of Shallow Foundation

## 6<sup>th</sup> Semester

Compiled by

Prof Dr. Irshad Ahmad

Department of Civil Engineering  
University of Engineering & Technology  
Peshawar

©Irshad 2013

## Table of Contents

Table of Contents .....	ii
Chapter 2 Bearing Capacity of Shallow Foundation.....	4
2.1 (Ultimate) Bearing Capacity ( $q_{ult}$ ).....	4
2.2 MODES OF SHEAR FAILURE.....	4
2.2.1 General Shear Failure (figure 2.1a).....	4
2.2.2 Local Shear Failure (figure 2.1 b).....	4
2.2.3 Punching Shear Failure (figure 2.1c).....	4
2.3 Gross Pressure Intensity ( $q_{gross}$ ):- .....	5
2.3.1 Net Pressure Intensity ( $q_{net}$ ).....	6
2.4 Safe bearing capacity ( $q_{safe}$ ) .....	7
2.5 Allowable Bearing Capacity ( $q_a$ ).....	7
2.6 Methods of bearing capacity determination.....	7
2.7 Analytical Methods.....	7
2.7.1 Bound Methods.....	8
2.7.2 Lower Bound Theorem .....	8
2.7.3 Upper Bound Theorem.....	10
2.7.4 Limit Equilibrium Method .....	11
2.8 Terzaghi's Bearing Capacity Equation (1943).....	13
EXAMPLE PROBLEM: Using the above problem statement, find Terzaghi bearing capacity factors ( $N_c$ , and $N_q$ ) using first principle. ....	19
EXAMPLE PROBLEM:.....	20
2.9 Effects of Water Table on Bearing Capacity of Soil.....	19
2.10 Meyerhof (1963) Bearing Capacity Equation.....	20
2.10.1 Bearing Capacity Factors ( $N_c$ , $N_q$ , $N_\gamma$ ) .....	21
2.10.2 Shape factor ( $s_c$ , $s_q$ , $s_\gamma$ ).....	21
2.10.3 Depth factors ( $d_c$ , $d_q$ , $d_\gamma$ ).....	21
2.10.4 Inclination factors ( $i_c$ , $i_q$ , $i_\gamma$ ):.....	21
2.10.5 Difference between Terzaghi's and Meyerhof's approach:-.....	22
2.10.6 Uses of B.C. equations.....	22
Terzaghi's equation:.....	22

Uses of Meyerhof Equation .....	22
2.10.7 Additional consideration in bearing capacity use:- .....	22
2.10.8 General Observation about Bearing Capacity Equation .....	23
2.10.9 Footing with Eccentric Loading.....	23
2.10.10 Alternative approach to consider eccentric loading .....	26
2.11 Behavior of clays under drained and undrained loading.....	28
2.11.1 Consolidated Undrained (CU) Test.....	28
2.11.2 Consolidated drained (CD) test.....	29
2.12 Total Vs Effect Stress Analyses.....	30
2.12.1 Sand.....	30
2.12.2 Clays .....	30

## CHAPTER 2

### BEARING CAPACITY OF SHALLOW FOUNDATION

#### 2.1 (Ultimate) Bearing Capacity ( $q_{ult}$ )

It is the least pressure which would cause shear failure of the supporting soil immediately below and adjacent to a foundation. What are the different shear failure modes of supporting soil?

#### 2.2 MODES OF SHEAR FAILURE

There are three modes of shear failures i.e. General, Local, and Punching shear failures depending upon the compressibility of soil and depth of footing with respect to its breadth (i.e. D/B ratio).

##### 2.2.1 General Shear Failure (figure 2.1a)

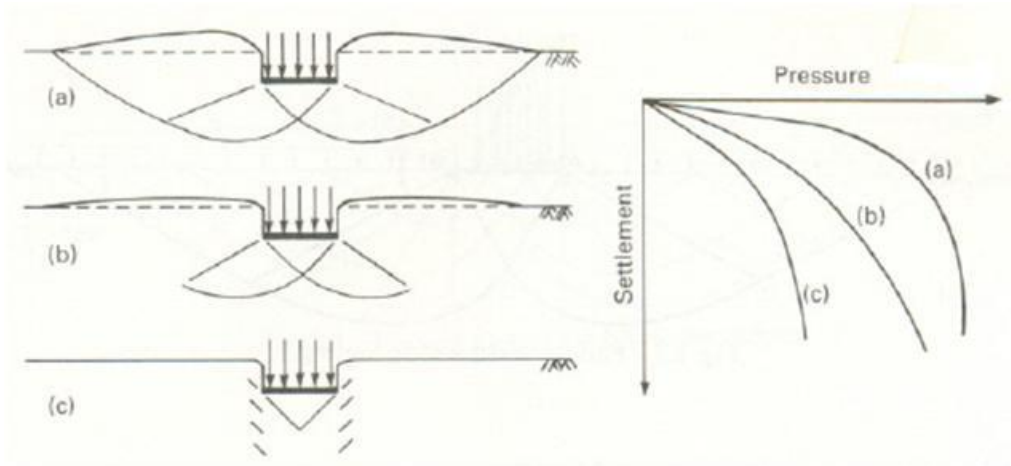
- Characterized by well defined failure pattern, consisting of a wedge and slip surface and bulging (heaving) of soil surface adjacent to the footing.
- Sudden collapse occurs, accompanied by tilting of the footing
- Occurs in dense or stiff soil.
- Failure load is well defined.

##### 2.2.2 Local Shear Failure (figure 2.1 b)

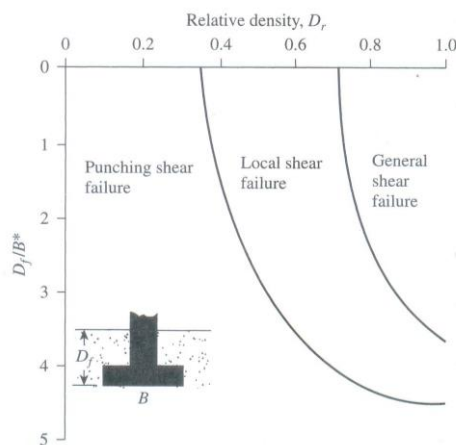
- Failure pattern consist of wedge and slip surface but is well defined only under the footing. Slight bulging of soil surface occurs. Tilting of footing is not expected.
- Large settlement occurs.
- Ultimate load is not well defined.
- Occurs in soil of high compressibility.

##### 2.2.3 Punching Shear Failure (figure 2.1c)

- Failure pattern is not well defined.
- No bulging of ground surface, no tilting of footing.
- Failure take place immediately below footing and surrounding soil remains relatively unaffected.
- Large settlements-ultimate load is not well defined.
- Occurs in soil of very high compressibility.
- It also occurs in the soil of very high compressibility, if the foundation is located at considerable depth (figure 2.2).



**Figure 2-1 Modes of Failures (a) general shear (b) local shear (c) punching shear**



**Figure 2-2 Effect of D/B and Dr on mode of failure**

The applied load (stress) causing shear failure of supporting soil can be in terms of gross or net pressure intensity.

### **2.3 Gross Pressure Intensity ( $q_{gross}$ ):-**

It is the total pressure at base of the footing due to the weight of superstructure and earth fill if any (figure 2.3)

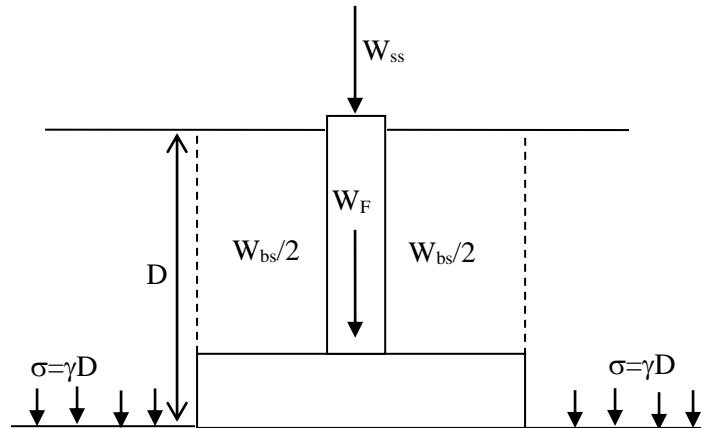
$W_{ss}$  = Load from superstructure.

$W_F$  = Weight of foundation.

$W_{bs}$  = Weight of the back fill soil.

$$q_{gross} = (W_{ss} + W_F + W_{bs})/A$$

A=Area of the footing



**Figure 2-3 Gross and Net pressure demonstration**

### 2.3.1 Net Pressure Intensity ( $q_{net}$ )

It is the increase in pressure at foundation level, being the total weight less the weight of the soil permanently removed.

(1) Before removal of soil, stress at foundation level is

$$= \gamma \times D$$

(2) After removal

$$q_{net} = q_{gross} - \gamma D$$

If  $q_{gross} = \gamma D$

$q_{net} = 0$  (it means that the weight of the soil excavated is equal to the weight of the structure)

Settlement of foundation (theoretically) = 0

Putting the relation for  $q_{gross}$

$$q_{net} = (W_{ss} + W_F + W_{bs})/A - \gamma D$$

$$q_{net} = (W_{ss})/A + (W_F + W_{bs})/A - \gamma D$$

if  $W_F$  is taken roughly equal to  $W_{bs}$  then

$$(W_F + W_{bs})/A = \gamma D$$

This leads to  $q_{net} = W_{ss}/A$

## 2.4 Safe bearing capacity ( $q_{\text{safe}}$ )

The safe bearing capacity (gross) to avoid shear failure is obtained by reducing (or dividing) the ultimate bearing capacity by a safety factor.

$$q_{\text{safe}} = q_{\text{ult}}/\text{FOS}$$

$$\text{FOS} = 2.5 \rightarrow 3 \text{ (Generally)}$$

It is not only the strength criteria that should put a limit on the applied stress, but the serviceability criteria (settlement of foundation) should also be considered.

$$q_{\text{safe}(\text{net})} \text{ in terms of net pressure is be in terns} = q_{\text{ult}}(\text{net}) / \text{FOS} = (q_{\text{ult}} - \gamma D) / \text{FOS}$$

## 2.5 Allowable Bearing Capacity ( $q_a$ )

It is the maximum pressure which may be applied to the soil such that the two fundamental requirements are satisfied.

- a) Limiting the settlement to a tolerable amount
- b) Shear failure of supporting soil is prevented.

So the allowable pressure is the minimum of

→  $q_{\text{safe}}$

→ Stress required to cause a specified amount of settlement

## 2.6 Methods of bearing capacity determination

- 1) Analytic method i.e. through bearing capacity equations like using Terzaghi equation, Meyerhof equation, Hansen equation etc.
- 2) Correlation with field test data e.g. Standard Penetration Test (SPT), Cone Penetration Test (CPT) etc.
- 3) On-site determination of bearing capacity e.g. plate load test (PLT), pile load test.
- 4) Presumptive bearing capacity (recommended bearing capacity in various codes)

We will discuss only Analytical Methods (1) in this chapter. Methods (2), (3) and (4) will be discussed in chapter-3.

## 2.7 Analytical Methods

Solutions for problems in mechanics must satisfy the three conditions of equilibrium, compatibility, and material properties. The complete solutions satisfying these conditions are very difficult even for very simple foundations and slopes. Therefore the standard methods used in geotechnical engineering involve simplifications. There are two basic methods: the bound methods and the limit equilibrium method. Both methods require approximations and simplifications.

### 2.7.1 Bound Methods

It is possible to ignore some of the conditions of equilibrium or compatibility to estimate the collapse load making use of important theorems of plastic collapse. If compatibility condition is satisfied, and equilibrium is ignored (Upper bound theorem), an upper bound to the true collapse load is obtained. If equilibrium is satisfied but compatibility is ignored (Lower bound theorem), a lower bound to the true load is obtained. These two theorems are applied to  $\phi=0$  soil in the next section.

Upper bound and lower bound solutions may give the exact solution for a problem if they match.

### 2.7.2 Lower Bound Theorem

The Lower bound states that “if an equilibrium distribution of stress can be found which balance the applied load and nowhere exceeds the strength of the soil (i.e. does not violate the yield criteria), the soil mass will not fail or will just be at a point of failure. It will be a lower bound estimated of capacity because a more efficient stress distribution may exist, which would be in equilibrium with higher external loads.

To calculate a lower bound, we must satisfy the conditions of equilibrium and the material properties (which determine the strength), but nothing is said about displacement or compatibility. The structure with a lower bound cannot collapse this is often known as the safe load.

We will now obtain a lower bound solution for the strip footing shown in figure 2.4 for  $\phi=0$  soil.

Consider equilibrium conditions in soil under the footing load. When the foundation pushes into the ground, stress block 1 has principal stresses as shown. The push into the ground however, displaces the soil on the right side of the line OY laterally, resulting in the major principal stress on block 2 being horizontal as shown. When the two blocks are adjacent to each other at the vertical line OY, then

$$\sigma_{3,1} = \sigma_{1,2}$$

We know that

$$\sigma_{1,1} = \sigma_{3,1} \times \tan^2\left(45 + \frac{\phi}{2}\right) + 2c \tan\left(45 + \frac{\phi}{2}\right)$$

$$\phi = 0 \Rightarrow \tan^2(45) = 1 \tag{2.1}$$

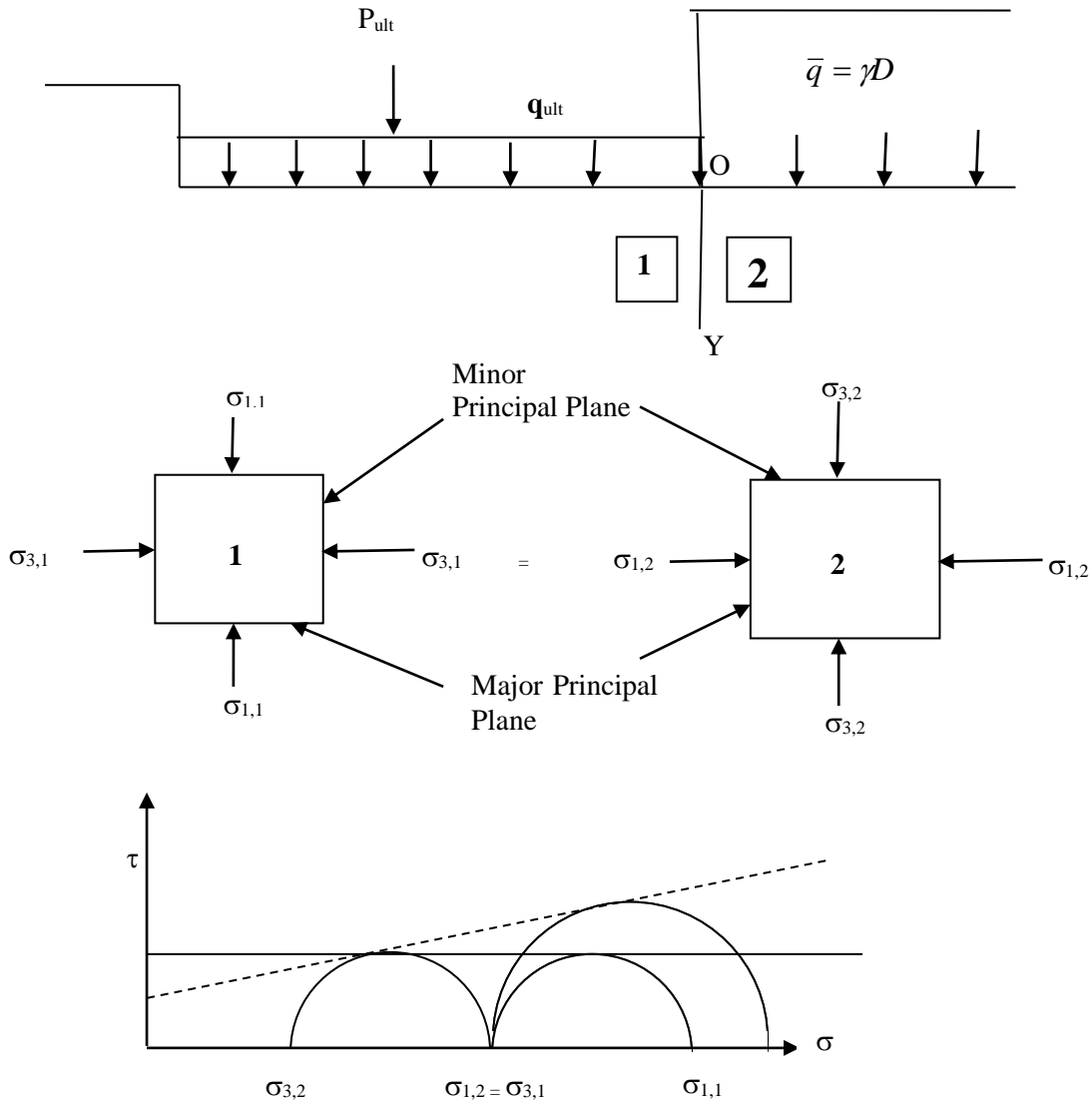
$$\sigma_{1,1} = \sigma_{3,1} + 2c$$

For block 2 at point O (corner of footing)

$$\sigma_{3,2} = \bar{q} = \gamma D \tag{2.2}$$



$$\sigma_{1,2} = \sigma_{3,2} \times \tan^2\left(45 + \frac{\phi}{2}\right) + 2c \tan\left(45 + \frac{\phi}{2}\right) \quad (2.3)$$



**Figure 2-4 Lower Bound solution for  $\phi=0$  soil**

Putting Eq-2.2 in Eq-2.3 and noting that  $\tan(45^\circ)=1$ .

$$\sigma_{1,2} = \sigma_{3,2} + 2c = \gamma D + 2c = \bar{q} + 2c \quad (2.4)$$

Now using equilibrium condition  $\sigma_{3,1}$  has to be equal to  $\sigma_{1,2}$  i.e.

$$\sigma_{3,1} = \sigma_{1,2}$$

Now using this equilibrium relationship in Eq-2.1 we get

$$\sigma_{1,1} = \gamma D + 2c + 2c = \gamma D + 4c = \bar{q} + 4c = q_{ult} \quad (2.5)$$

$$q_{ult} = \sigma_{1,1} = \bar{q} + 4c$$

### 2.7.3 Upper Bound Theorem

If you take any compatible mechanism (i.e. the motion of the sliding soil mass must be compatible with its continuity and with any boundary restrictions), and consider an increment of movement and if you show that the work done by the stresses in the soil equals the work done by the external loads, the structure must collapse. The external loads are an upper bound to the true collapse load because a more efficient mechanism may exist resulting in collapse under lower external loads.

To calculate an upper bound you must satisfy the conditions of compatibility and the material properties (which governs the work done by the stresses in the soil), but nothing is said about equilibrium. Because the structure with an upper bound load must collapse this is often known as the unsafe load.

Consider the case below of a circular slip surface. Consider an increment of movement  $d\theta$ . this will cause internal forces and external forces acting on the system to do some internal and external work respectively.

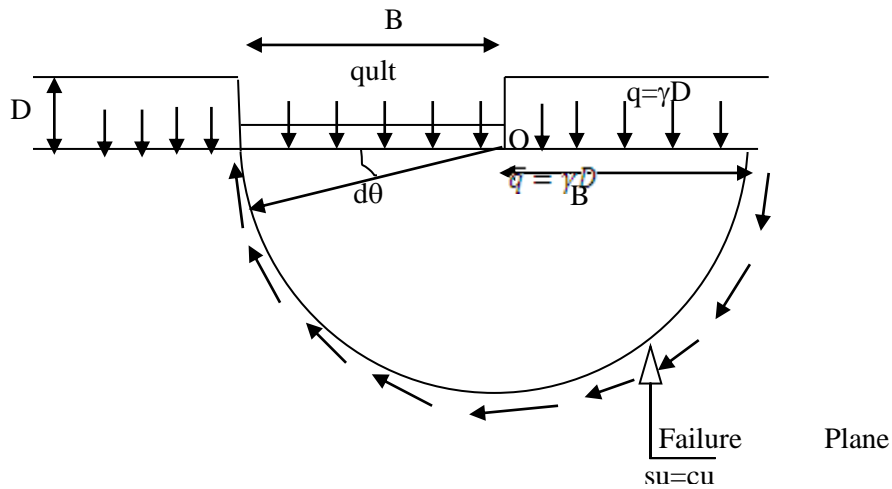
$$\Delta s = B \times d\theta$$

$$\text{Internal work } W_{int} = cu \times \pi B \times \Delta s = cu \times \pi B \times B \times d\theta$$

$$\text{External work } W_{ext} = (q_u \times B \times \Delta s/2) - (q \times B \times \Delta s/2) = q_u \times B \times B \times d\theta/2 - q \times B \times B \times d\theta/2$$

$$W_{int} = W_{ext}$$

$$q_u = 2\pi cu + q$$



### 2.7.4 Limit Equilibrium Method

The limit equilibrium method is by far the most commonly used analysis for the stability of geotechnical structures. The steps in calculating a limit equilibrium solution are as follows:

1. Draw an arbitrary collapse mechanism of slip surfaces; this may consist of any combination of straight lines or curves arranged to give a mechanism.
2. Calculate the statical equilibrium of the components of the mechanism by resolving forces or moments and hence calculate the external forces or the strength mobilized in the soil (whichever is unknown).
3. Examine the statical equilibrium of other mechanisms and so find the critical mechanism for which the loading is the limit equilibrium load.

Limit equilibrium and upper bound can, in certain cases, be identical because they require an assumption of kinematic failure mechanism.

Application of Limit equilibrium method to the same problem as above is given below:

$$\sum M_o = 0$$

$$q_{ult} \times B \times \frac{B}{2} - \bar{q} \times B \times \frac{B}{2} - c \times B \times B = 0$$

$$q_{ult} = \frac{c \times B + \bar{q} \times \frac{B}{2}}{\frac{B}{2}}$$

$$(q_{ult} \times B) \times \frac{B}{2} - (\bar{q} \times B) \times \frac{B}{2} - \tau \times \Pi \times B^2 = 0$$

$$B^2 \left( \frac{q_{ult}}{2} - \frac{\bar{q}}{2} - \tau \times \Pi \right) = 0$$

$$q_{ult} = 2\Pi\tau + \bar{q} \Rightarrow q_{ult} = 2c\Pi + \bar{q} (\because \tau = c)$$

$$q_{ult} = \bar{q} + 2c\Pi \text{ Upper bound solution (U.B.S)}$$

$$q_{ult} = \bar{q} + 4c \text{ Lower bound solution (L.B.S)}$$

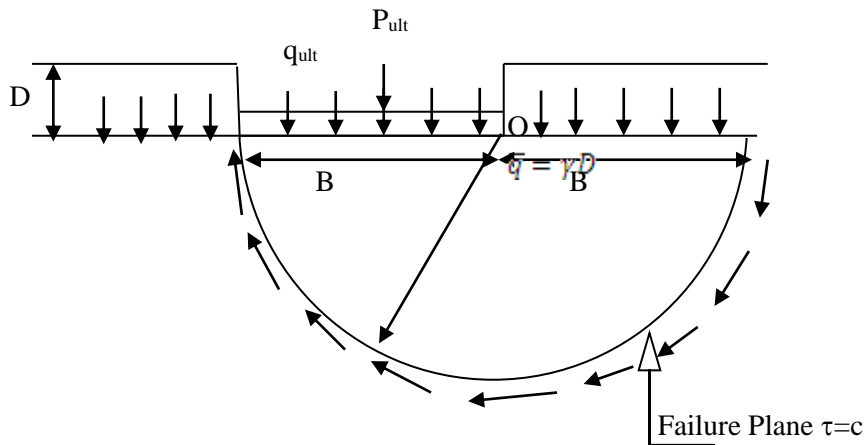


Figure 2-5 Upper bound theorem

For surfacing footing;

$$\bar{q} = 0$$

$$q_{ult} = 2\Pi c \rightarrow U.B.S$$

$$q_{ult} = 4c \rightarrow L.B.S$$

Average of

$$q_{ult} = \frac{2\Pi c + 4c}{2} = 5.14c$$

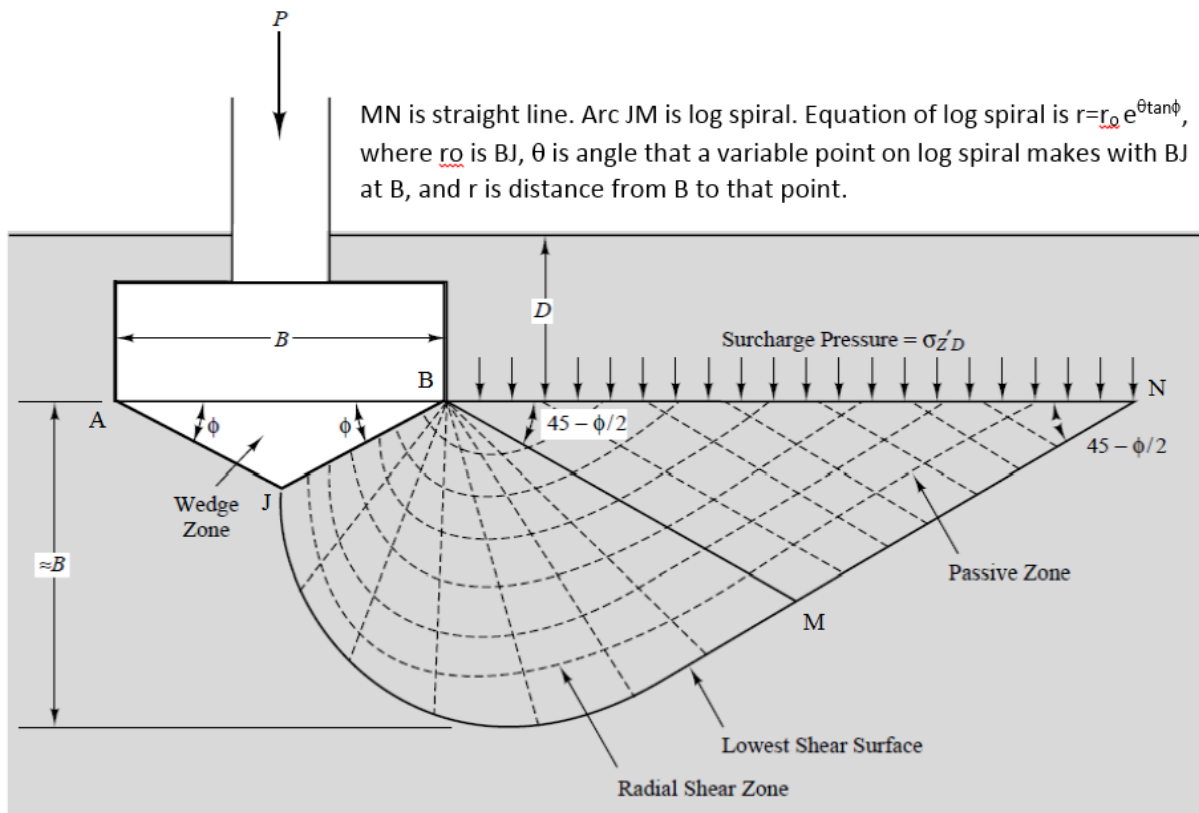
## 2.8 Terzaghi's Bearing Capacity Equation (1943)

Terzaghi developed a general formula for ultimate bearing capacity of spread footing foundations using the Limit Equilibrium method. He made the following assumptions:

- The depth of the footing is less than or equal to its width (D, B).
- The foundation is rigid and has a rough bottom.
- The soil beneath the footing is a homogeneous semi-infinite mass.
- Strip foundation with a horizontal base and level ground surface under vertical loads.
- The general shear mode of failure governs and no consolidation of the soil occurs (settlement is due only to shearing and lateral movement of the soil).
- The shear strength of the soil is described by  $s = c + \sigma \tan \phi$

The collapse mechanism assumed by Terzaghi is given in figure 2-6. Terzaghi considered three zones in the soil, as shown in Figure 6.5. Immediately beneath the foundation is a (Elastic) wedge zone that remains intact and moves downward with the foundation. The movement of the wedge forces the soil aside and produces radial shear zone and linear shear zone. The radial shear zone extends from each side of the wedge, where he took the shape of the shear planes to be logarithmic spirals. The outer portion is the linear shear or Passive zone in which the soil shears along planar surfaces. Since Terzaghi neglected the shear strength of soils between the ground surface and a depth D, the shear surface stops at this depth and the overlying soil has been replaced with the surcharge pressure  $q = \gamma D$ . This approach is conservative, and is part of the reason for limiting the method to relatively shallow foundations ( $D \sim B$ ).

Terzaghi developed his theory for continuous foundations (i.e., those with a very large L/B ratio). This is the simplest case because it is a two-dimensional problem. He then extended it to square and round foundations by adding empirical coefficients (shape factors) obtained from model tests.



**Figure 2-6 Collapse Mechanism assumed by Terzaghi (Only right side of the slip lines/failure mechanism is shown in the figure. Failure mechanism is symmetrical)**

The free body diagram of elastic wedge is shown in figure 2-7.

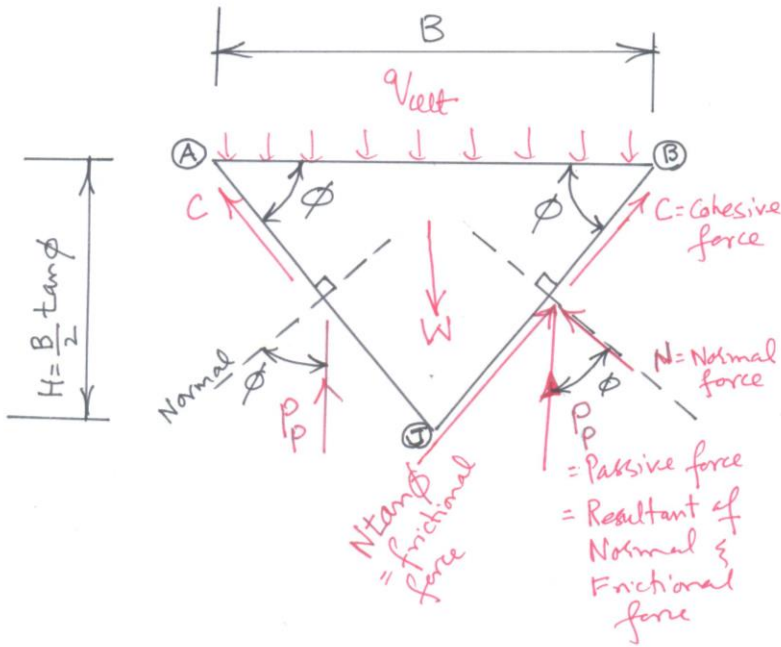


Figure 2-7

$$\sum F_y = 0$$

$$q \times B = -W + 2C \sin \phi + 2P_p$$

$$C = c \times JB \quad [c = \text{cohesion of soil}]$$

$$C = c \times \frac{B}{2 \cos \phi}$$

$$W = \gamma \times \frac{B \times H}{2} = \frac{\gamma B}{2} \left( \frac{B}{2} \tan \phi \right)$$

$$W = \frac{\gamma B^2}{4} \tan \phi$$

Contribution to  $P_p$  is due to the self weight of the soil  $\gamma$ , soil cohesion  $c$ , and surcharge  $q = \gamma D$ . Therefore  $P_p$  is divided respectively into  $P_{p\gamma}$ ,  $P_{pc}$ , and  $P_{pq}$ .

However finding all three components of  $P_p$  simultaneously is an indeterminate problem. To remedy this difficulty, we split the problem into three pieces.

The three separate problems are defined as follows:

Problem 1: Evaluate  $P_{pc}$  by assuming the soil has cohesion and friction but is weightless and has no surcharge.

Problem 2: Evaluate  $P_p$  by assuming the soil has surcharge and friction but has no cohesion and is weightless.

Problem 3: Evaluate  $P_p$  by assuming the soil has weight and friction but no cohesion and no surcharge.

This method of superposition introduces errors but the simplification is conservative and does not seem to introduce major error.

After evaluating these components of  $P_p$  (not done here), and putting their values in the above equation of equilibrium, the Terzaghi bearing capacity equation is obtained.

$$q_{ult} = cN_c + qN_q + 0.5 \gamma B N_\gamma$$

$N_c$ ,  $N_q$  and  $N_\gamma$  are bearing capacity factors or coefficients due to cohesion, surcharge and soil weight respectively. They depend on the value of  $\phi$  and on the shape of the failure zone as assumed by the different researchers.

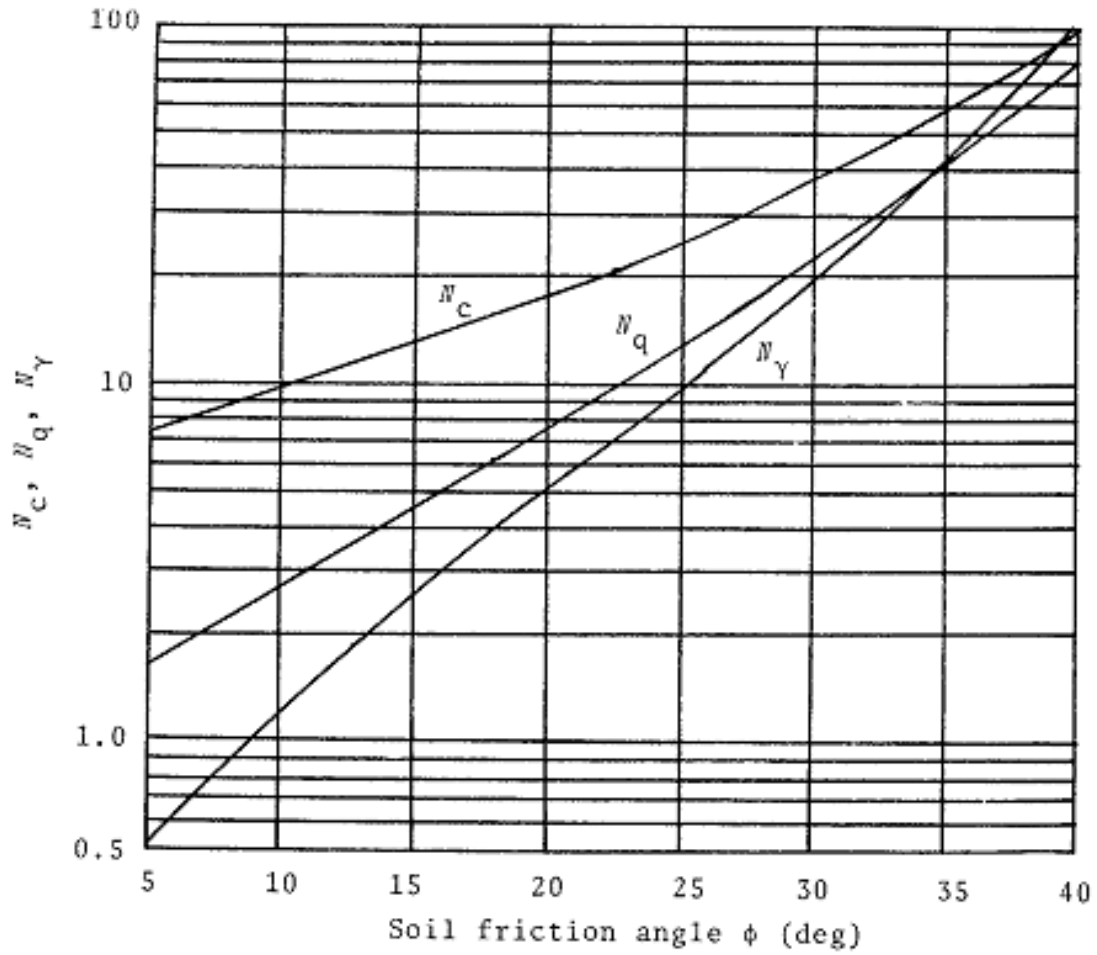
Terzaghi used shape factors to make the formula applicable to other shapes of foundations using the shape factors  $s_c$  and  $s_\gamma$ .

$$q_{ult} = cN_c s_c + qN_q + 0.5 \gamma B N_\gamma s_\gamma$$

The first term in the BC equation is the contribution to BC due to cohesion of soil, the 2<sup>nd</sup> term corresponds to the overburden pressure or depth of the footing, the 3<sup>rd</sup> term is due to the self-weight of the soil.

Shape factor	Strip footing	Round	Square	Rectangular
$S_c$	1	1.3	1.3	$1 + \left(\frac{B}{L}\right) \frac{N_q}{N_c}$
$S_\gamma$	1	0.6	0.8	$1 - 0.4 \frac{B}{L}$





**Figure 2-8 Terzaghi's Bearing Capacity factors ( $N_c$ ,  $N_q$ ,  $N_\gamma$ )**

## Bearing-capacity factors for the Terzaghi equations

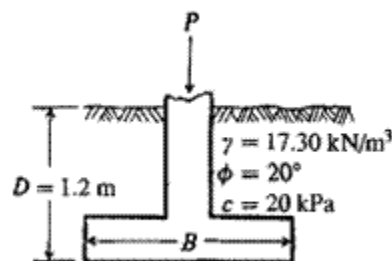
$\phi$ , deg	$N_c$	$N_q$	$N_\gamma$
0	5.7*	1.0	0.0
5	7.3	1.6	0.5
10	9.6	2.7	1.2
15	12.9	4.4	2.5
20	17.7	7.4	5.0
25	25.1	12.7	9.7
30	37.2	22.5	19.7
34	52.6	36.5	36.0
35	57.8	41.4	42.4
40	95.7	81.3	100.4
45	172.3	173.3	297.5
48	258.3	287.9	780.1
50	347.5	415.1	1153.2

### EXAMPLE PROBLEM:

Compute the allowable bearing pressure using the Terzaghi equation for the square footing of width  $B=1.5\text{m}$  shown in figure below. The soil data are obtained from a series of undrained U triaxial tests. Is the soil saturated?

#### *Solution.*

1. The soil is not saturated, since a U test gives a  $\phi$  angle. A CU test might give similar data for a saturated soil.



**EXAMPLE PROBLEM:** Using the above problem statement, find Terzaghi bearing capacity factors ( $N_c$ , and  $N_q$ ) using first principle.

## 2.9 Effects of Water Table on Bearing Capacity of Soil

Effective unit weight of the soil is used in the bearing capacity equations for computing the ultimate bearing capacity. The effective unit weight of soil should be used in accordance with the table given below.

No	Position of water table	2nd Term	3rd Term
1	Ground surface	$\gamma' DN_q; \gamma' = \gamma_{sat} - \gamma_w$	$1/2 \gamma' BN_r; \gamma' = \gamma_{sat} - \gamma_w$
2	If the water table is at footing level or base of footing.	$\gamma$	$\gamma' = \gamma_{sat} - \gamma_w$
3	Water table below the wedge	$\gamma' = \gamma$	$\gamma' = \gamma$
4	Water table between 1 and 2	$\gamma' = \frac{d_1 \gamma + d_2 (\gamma_{sat} - \gamma_w)}{D}$	$\gamma' = \gamma_{sat} - \gamma_w$
5	Water table at depth $Z_w$ from the base of footing.	$\gamma$	$\gamma' = (\gamma_{sat} - \gamma_w) + \frac{Z_w}{H} (\gamma - \gamma_{sub})$

$H = B/2 \times \tan(45 + \phi/2)$ ,  $\gamma'$  = Bulk unit weight,  $\gamma_{sat}$  = saturated unit weight,  $\gamma_{sub}$  = submerged unit weight =  $\gamma_{sat} - \gamma_{water}$

**Note:** For drained conditions, where effective shear strength parameters are used for bearing capacity calculations, the effect of water table should be considered.

For undrained conditions, total shear strength parameters are used. Therefore total unit weight is used in the bearing capacity equations and no effect of water table effect is considered.

### EXAMPLE PROBLEM

A footing  $2.5 \times 2.5$  m carries a pressure of  $400 \text{ kN/m}^2$  at a depth of 1 m in a sand. The saturated unit weight of the sand is  $20 \text{ kN/m}^3$  and the unit weight above the water table is  $17 \text{ kN/m}^3$ . The design shear strength parameters are  $c' = 0$  and  $\phi' = 40^\circ$ . Determine the factor of safety with respect to shear failure for the following cases:

- the water table is 5 m below ground level,
- the water table is 1 m below ground level,
- the water table is at ground level and there is seepage vertically upwards under a hydraulic gradient of 0.2.

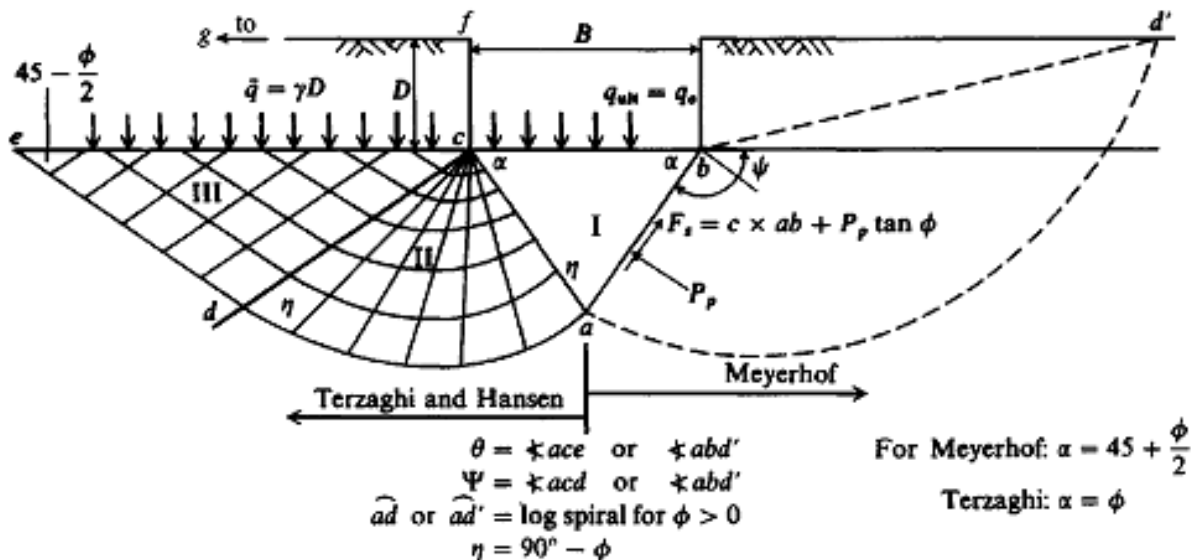
### EXAMPLE PROBLEM:

A strip footing is to be designed to support a dead load of 500 kN/m and an imposed load of 300 kN/m at a depth of 0.7 m in a gravelly sand. Characteristic values of the shear strength parameters are  $c' = 0$  and  $\phi' = 40^\circ$ . Determine the required width of the footing if a (lumped) factor of safety of 3.0 against shear failure is specified and assuming that the water table may rise to foundation level.

The unit weight of the sand above the water table is  $17 \text{ kN/m}^3$  and below the water table the saturated unit weight is  $20 \text{ kN/m}^3$ .

### 2.10 Meyerhof (1963) Bearing Capacity Equation

A comparison of the Failure Mechanism assumed by Meyerhof and Terzaghi is given below.



Meyerhof bearing capacity equation is given below. It takes into account the reduction caused by moment/eccentricity of load on the footing, and also reduction in bearing capacity due to inclination of load on the footing.

$$q_{ult} = cN_c d_c s_c i_c + \gamma' DN_q d_q s_q i_q + \frac{1}{2} \gamma' B' N_r d_r s_r i_r$$

$\gamma'$  is effective unit weight of the soil to take into account the water table effect.

$B'$  is the effective width of footing to consider the effect of moment as will be discussed later

### 2.10.1 Bearing Capacity Factors ( $N_c$ , $N_q$ , $N_\gamma$ )

$$N_q = e^{\pi \tan \phi} \tan^2 \left( 45 + \frac{\phi}{2} \right)$$

$$N_c = (N_q - 1) \cot \phi$$

$$N_r = (N_q - 1) \tan(1.4\phi)$$

### 2.10.2 Shape factor ( $s_c$ , $s_q$ , $s_\gamma$ )

$$s_c = 1 + 0.2K_p \frac{B'}{L'} \quad \text{for any } \phi$$

$$s_q = s_\gamma = 1 + 0.1K_p \frac{B'}{L'} \quad \text{for } \phi > 10^\circ \quad (\text{For any } \phi)$$

$$s_q = s_\gamma = 1 \quad \text{for } \phi = 0^\circ$$

### 2.10.3 Depth factors ( $d_c$ , $d_q$ , $d_\gamma$ )

$$d_c = 1 + 0.2\sqrt{K_p} \frac{D}{B'} \quad (\text{For any } \phi)$$

$$d_c = d_q = 1 + 0.1\sqrt{K_p} \frac{D}{B'} \quad (\phi > 10^\circ)$$

$$d_q = d_r = 1 \quad \phi = 0^\circ$$

where

$$K_p = \tan^2 \left( 45 + \frac{\phi}{2} \right)$$

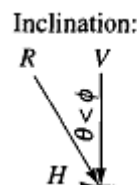
### 2.10.4 Inclination factors ( $i_c$ , $i_q$ , $i_\gamma$ ):

The effect of load inclination is to reduce the bearing capacity of the soil.

**Inclined Loads.** In addition to the vertical load acting on the footing, it may also be subjected to a lateral load; hence the resultant of the load will be inclined. One possible method as proposed by Merhof is to reduce the allowable bearing capacity based on the inclination of the load. However, this approach has a drawback in that the geotechnical engineer usually does not know the inclination of the various loads when preparing the foundation report. And if the inclinations were known, then numerous allowable bearing capacities would be needed for the various inclinations of the load.

$$i_e = i_q = \left( 1 - \frac{\theta}{90} \right)^2$$

By Prof. Dr. Irshad Ahmad, 2014



$$i_r = \left(1 - \frac{\theta}{\phi}\right)^2 \quad \phi > 0^\circ$$

$$i_r = 0 \quad \text{For } \phi = 0^\circ$$

Where  $\theta$  is the angle of the resultant of load on the footing with the vertical.

### 2.10.5 Difference between Terzaghi's and Meyerhof's approach:-

- Difference between “N” factors exists because of assumption of log spiral “ad” and exit wedge “cde”.
- Meyerhof's shape factors do not differ greatly than those given by Terzaghi except for addition of “sq”.
- Meyerhof approximately accounted for shear along c`d` in his analysis. However observing that shear effect is still being ignored he introduced depth factor.
- For D is approximately equal to B and Meyerhof is approximately equal to Terzaghi, but the difference become pronounced for larger D/B ratio.

### 2.10.6 Uses of B.C. equations

#### **Terzaghis equation:**

- Very cohesive soil when  $D/B \leq 1$ .
- quick estimate of the  $q_{ult}$ .
- Do not use for footings with horizontal forces, for tilted base, for sloping ground.

#### **Uses of Meyerhof Equation**

- For any situation.

### 2.10.7 Additional consideration in bearing capacity use:-

- Do not interpolate “N” factors over about 2.
- For  $\phi > 35^\circ$ , N factors change rapidly and by large amounts.
- Bearing capacity equation tends to be conservative.
- Terzaghi developed bearing capacity equation for general shear failure. For local shear failure he proposed reducing c,  $\phi$  i.e.

$$c'' = \frac{2}{3}c \quad \text{and} \quad \phi'' = \tan^{-1}\left(\frac{2}{3}\tan\phi\right)$$

- The 3<sup>rd</sup> term with  $N\gamma$  in bearing capacity equations do not increase without bound. Use reduction factor with the term. [Note: We have not considered this factor in our course, so you need not to use it]

$$\gamma_r = 1 - 0.25 \log\left(\frac{B}{R}\right)$$

- If B is greater than or equal to 2m (6ft) then R=2.0 for SI units; R=6 for fps units. So the term will be  $\left(\frac{1}{2}\gamma B N_r S_r d_r i_r\right)(\gamma_r)$  for Meyerhof equation and  $\left(\frac{1}{2}\gamma B N_r S_r\right)(\gamma_r)$  for Terzaghi's equation.

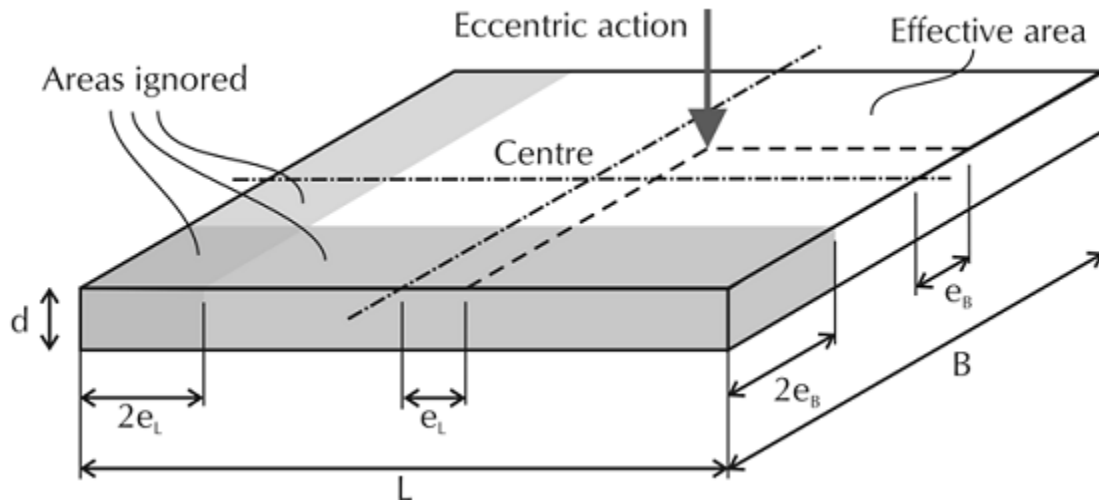
### 2.10.8 General Observation about Bearing Capacity Equation

- The cohesive term  $cN_c$  predetermines in cohesive soils.
- The depth or overburden term “ $qN_q$ ” predetermines in cohesion less soil.
- The self weight or breadth term “ $\frac{1}{2}\gamma B N_r$ ” provides some increase in bearing capacity both for cohesive and cohesion less soil. For  $B < 3$  to 4m it can even be neglected.
- No one would place footing on the surface of cohesion less soil.

### 2.10.9 Footing with Eccentric Loading

A footing may be eccentrically loaded from a concentric column with an axial load and moments about one or both axes. The eccentricity may result also from a column that is initially not centrally located or becomes off-center when a part of the footing is cut away during remodeling and/or installing new equipment. Obviously the footing cannot be cut if an analysis indicates the recomputed soil pressure might result in a bearing failure.

The eccentricity effect is taken into account by using a reduced width of footing called “effective width”  $B'$  in the last term of the bearing capacity equation.



For a footing subjected to a vertical load  $V$ , moment  $M_B$  and moment  $M_L$ , the eccentricities  $M_L$ , the eccentricities  $e_B = M_B/V$  and  $e_L = M_L/V$ .

Effective width =  $B'$

$$B' = B - 2e_B$$

Effective length =  $L'$

$$L' = L - e_L$$

$e \leq B/6$  ( $e$  is usually limited to  $B/6$ )

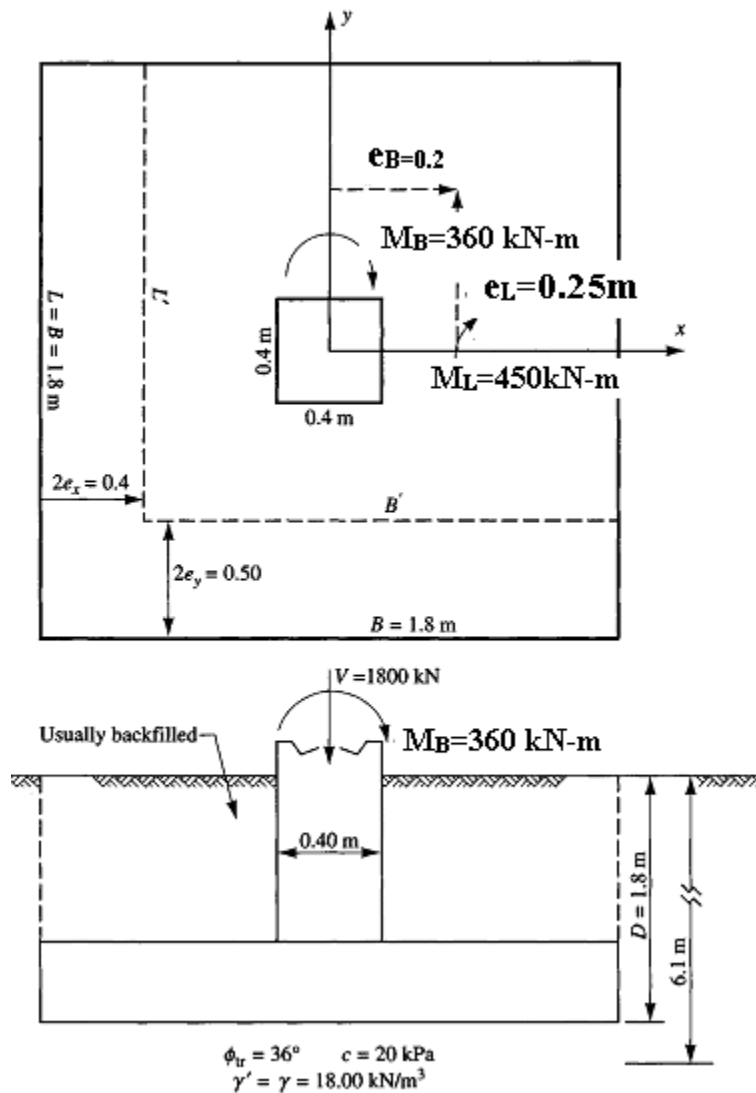
Use  $B'$  and  $L'$  in Meyerhofs shape factor, depth factor and last term of bearing capacity.

Effective Area,  $A_f = B' \times L'$

### EXAMPLE PROBLEM

What is the allowable soil pressure (FOS=3) using Meyerhof's Bearing capacity equation.



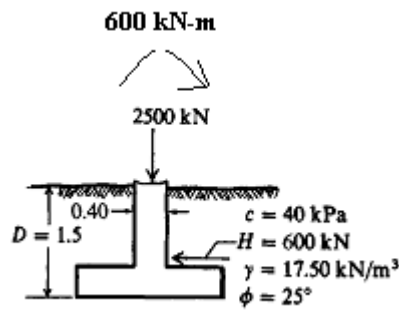


### EXAMPLE PROBLEM

The base of a long retaining wall is 3 m wide and is 1 m below the ground surface in front of the wall: the water table is well below base level. The vertical and horizontal components of the base reaction are 282 and 102 kN/m, respectively. The eccentricity of the base reaction is 0.36 m. Appropriate shear strength parameters for the foundation soil are  $c' = 0$  and  $\phi' = 35^\circ$ , and the unit weight of the soil is  $18 \text{ kN/m}^3$ . Determine the factor of safety against shear failure.

### EXAMPLE PROBLEM

Find the size of rectangular footing using Meyerhof's equation. The footing is subjected to both horizontal and moment as shown in the figure.



### 2.10.10 Alternative approach to consider eccentric loading

Usually at the time of geotechnical investigation, the eccentricity of load is not known. In this case, allowable bearing capacity is recommended without considering moment/eccentricity effects. Later on, at the time of design of footing, the footing pressure is checked.

Because an eccentrically loaded footing will create a higher bearing pressure under one side as compared to the opposite side, one approach is to evaluate the actual pressure distribution beneath the footing. The usual procedure is to assume a rigid footing (hence linear pressure distribution) and use the section modulus ( $\frac{1}{6} B^2$ ) in order to calculate the largest and lowest bearing pressure. For a footing having a width  $B$ , the largest ( $q'$ ) and lowest ( $q''$ ) bearing pressures are as follows:

$$q' = \frac{Q(B+6e)}{B^2}$$

$$q'' = \frac{Q(B-6e)}{B^2}$$

where  $q'$  = largest bearing pressure underneath the footing, which is located along the same side of the footing as the eccentricity (psf or kPa)

$q''$  = lowest bearing pressure underneath the footing, which is located at the opposite side of the footing (psf or kPa)

$Q = P$  = vertical load applied to the footing (pounds per linear foot of footing width or kN per linear meter of footing width)

$e$  = Eccentricity of the load  $Q$ , i.e., the lateral distance from  $Q$  to the center of gravity of the footing (ft or m)

$B$  = width of the footing (ft or m)

A usual requirement is that the load  $Q$  must be located within the middle one-third of the footing and the above equations are only valid for this condition. The value of  $q'$  must not exceed the allowable bearing pressure  $q_{all}$ .

### EXAMPLE PROBLEM

A strip footing will be constructed on a nonplastic silty sand deposit that has the shear strength properties (i.e.,  $c' = 0$  and  $\phi' = 30^\circ$ ) and a saturated unit weight of 125 pcf (19.7 kN/m<sup>3</sup>). The proposed strip footing will be 4 ft (1.2 m) wide and embedded 2 ft (0.6 m) below the ground surface. Use a factor of safety of 3 and use

Assume the groundwater table is located 4 ft (1.2 m) below ground surface. Determine the allowable bearing pressure  $q_{all}$  and the maximum vertical concentric load the strip footing can support for the nonplastic silty sand.

**use Terzaghi BC equation with Meyerhof's BC factors**

### EXAMPLE PROBLEM

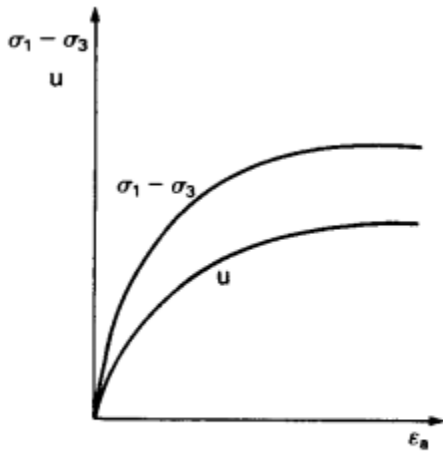
**Continued from previous example** Assume the vertical load exerted by the strip footing = 100 kN per linear meter of wall length and that this load is offset from the centerline of the strip footing by 0.15 m (i.e.,  $e = 0.15$  m). Determine the largest bearing pressure  $q'$  and the least bearing pressure  $q''$  exerted by the eccentrically loaded footing. Is  $q'$  acceptable from an allowable bearing capacity standpoint?

## 2.11 Behavior of clays under drained and undrained loading

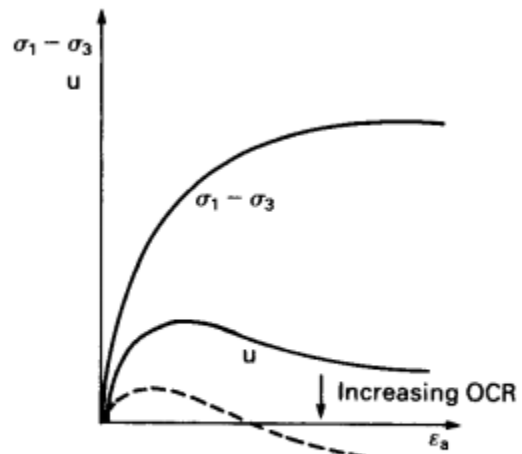
### 2.11.1 Consolidated Undrained (CU) Test

- Observations:
- (i) Failures occur at relatively large strains both in N.C.C. & O.C.C.
  - (ii) Since drainage is not allowed during  $\sigma_d$  application therefore "u" would develop in soil samples. & the volume change ( $\Delta V$ ) would be zero. of the soil sample.
  - (iii) ~~For N.C.C.~~ as  $\sigma_d$  is applied, the sample will be shortened i.e. it will show axial strain - ~~but~~ however, the sample will laterally bulge to have  $\Delta V = 0$ . It means that the axial shortening is accommodated by lateral bulging so that  $\Delta V = 0$ .
  - (iv) For N.C.C.; with the application of  $\sigma_d$ , the soil sample is trying to compress or reduce in volume but it cannot reduce in volume because drainage is prevented as a result, pore water pressure would develop in the sample.
  - (v) For O.C.C. - with greater O.C.R.; during  $\sigma_d$  application, the soil particles cannot get into more compact position, so the soil particles would try to roll one over the other i.e. the soil sample has the tendency to dilate

or increase in volume - but it cannot do so because:  
(8) no drainage ( $\Delta V = 0$ ); as a result the pore water pressure is -ive.

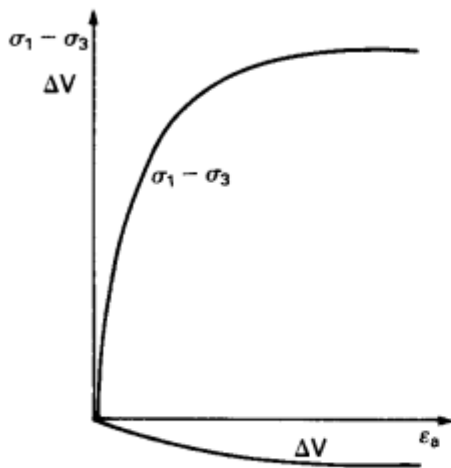


(a) Consolidated-undrained test normally consolidated clay

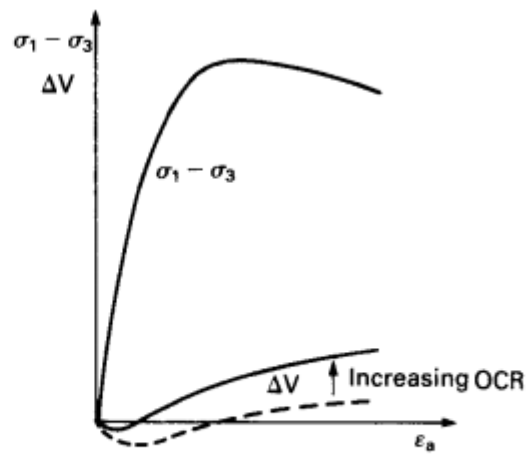


(b) Consolidated-undrained test overconsolidated clay

### 2.11.2 Consolidated drained (CD) test



(c) Drained test normally consolidated clay



(d) Drained test overconsolidated clay

### Observations.

- (i) This is drained test, hence " $u=0$ " &  $\Delta V$  will be 0.
- (ii) Failure in OCC occurs at low strain with definite peak of  $\sigma_d$ .
- (iii) In NCC, the soil sample is compressed &  $\Delta V$  decreases.
- (iv) In OCC, the soil particles cannot get into more compact form & thus the particles roll over one another leading to increase in volume "dilatation" after some initial compression.

## 2.12 Total Vs Effect Stress Analyses

### 2.12.1 Sand

Sand, because of its greater permeability, remains under drained condition. Under special conditions of loading, like the blast loading or earthquake loading, where the rate of loading is very fast, sand remains under undrained condition. To evaluate bearing capacity of footing resting on sand, drained conditions are assumed, and therefore effective shear strength parameters are used and the location of the groundwater table can not affect the ultimate bearing capacity.

The ultimate bearing capacity of plastic soil is often much less than the ultimate bearing capacity of cohesionless soil. This is the reason that building codes allow higher allowable bearing pressures for cohesionless soil (such as sand) than plastic soil (such as clay). Also, because the ultimate bearing capacity does not increase with footing width for saturated plastic soils, there is often no increase allowed for an increase in footing width.

### 2.12.2 Clays

For saturated plastic soils, the bearing capacity often has to be calculated for two different conditions:

1 Total stress analyses (short-term condition) that use the undrained shear strength of the plastic soil

2 Effective stress analyses (long-term condition) that use the drained shear strength parameters

( $c'$  and  $\phi'$ ) of the plastic soil

#### **Total Stress Analysis of clays**

The total stress analysis uses the undrained shear strength of the plastic soil. The undrained shear strength  $s_u$  could be determined from field tests, such as the vane shear test (VST), or in the laboratory from

unconfined compression tests, or unconsolidated undrained (UU) triaxial test, or consolidated undrained triaxial (CU) with shear strength parameters found in total stress.

**Application of shear strength parameters in terms of total stress obtained from CU test:** In some cases, it may be appropriate to use total stress parameters  $c$  and  $\phi$  in order to calculate the ultimate bearing capacity. For example, a structure (such as an oil tank or grain-elevator) could be constructed and then sufficient time elapses so that the saturated plastic soil consolidates under this load. If an oil tank or grain elevator were then quickly filled, the saturated plastic soil would be subjected to an undrained loading. This condition can be modeled by performing consolidated undrained triaxial compression tests (ASTM D 4767-02, 2004) in order to determine the total stress parameters ( $c$  and  $\phi$ ).

### Effective Stress Analysis of clays

The effective stress analysis uses the drained shear strength ( $c'$  and  $\phi'$ ) of the plastic soil. The drained shear strength could be obtained from triaxial compression tests with pore water pressure measurements performed on saturated specimens of the plastic soil. This analysis is termed a long-term analysis because the shear induced pore water pressures (positive or negative) from the loading have dissipated and the hydrostatic pore water conditions now prevail in the field. Because an effective stress analysis is being performed, the location of the groundwater table must be considered in the analysis.

### Soft Clays

For normally consolidated clays, the critical conditions are the short term conditions, in which the clays are under undrained condition. Total shear strength parameters should be used to evaluate bearing capacity with no effect of water table is considered. Long term conditions for NCC are not critical because with time the soil consolidates (the the excess pore water pressure dissipates and the effective stress increases), the soil shear strength and bearing capacity increases.

$$q_{ult} = c \times N_c + \gamma \times D \times N_q + 0.5 \times \gamma \times B \times N_\gamma$$

Considering  $c_u > 0$ , and  $\phi_u = 0$ , Terzaghi's BC factors are:  $N_c = 5.4$ ,  $N_q = 1$ ,  $N_\gamma = 0$

$$q_{ult} = 5.4 c_u + \gamma \times D$$

ultimate bearing capacity in terms of net pressure  $q_{ult}(\text{net}) = q_{ult} - \gamma \times D$

$$q_{ult}(\text{net}) = 5.4 c_u$$

if  $c_u$  is determined using unconfined compression test. Then  $c_u = q_u/2$ . Where  $q_u$  is unconfined compression strength

$$q_{ult}(\text{net}) = 5.4 \times q_u/2$$

considering FOS=3 against shear failure of the soil

$$q_{ult}(\text{net}) \cong q_u$$

that is ultimate bearing capacity in terms of net pressure can be considered equal to unconfined compression strength.

### Heavily overconsolidated Clays

Usually the effective stress analysis will provide a lower allowable bearing capacity for very stiff or hard saturated plastic soils. This is because such plastic soils are usually heavily overconsolidated and they tend to dilate (increase in volume) during undrained shear deformation. A portion of the undrained shear strength is due to the development of negative pore water pressures during shear deformation. As these negative pore water pressures dissipate with time, the shear strength of the heavily overconsolidated plastic soil decreases. For the long-term case (effective stress analysis), the shear strength will be lower resulting in a lower bearing capacity.

### **Not heavily OCC**

Firm to stiff saturated plastic soils are intermediate conditions. The overconsolidation ratio (OCR) and the tendency of the saturated plastic soil to consolidate (gain shear strength) will determine whether the short-term condition (total stress parameters) or the long-term condition (effective stress parameters) provides the lower bearing capacity.

### **EXAMPLE PROBLEM**

A strip footing will be constructed over heavily over-consolidated clay that has an undrained shear strength  $s_u = 200$  kPa (i.e.  $c_u=200$  kPa,  $\phi_u=0$ ), and a drained shear strength of  $\phi'=28^\circ$ ,  $c'=5$  kPa. The proposed strip footing will be 1.2m wide, and embedded 0.6m below the ground surface. Assume the W.T is located at a depth of 0.6m. The saturated unit weight of the clay is  $19.7$  kN/m<sup>3</sup> both above and below the W.T. Perform both a total stress analysis and an effective stress analysis, determine the allowable load using Terzaghi equation with Meyerhof BC factors.

## **2.13 Foundation Design Philosophy**

There are two possible approaches to foundation design, as follows

### **Allowable stress design (ASD)**

The method so far discussed in this chapter is based on ASD.



Most practising geotechnical engineers use the allowable stress design (ASD) method when designing shallow foundations. In this case, the allowable bearing capacity is given by:

$$q_{all} = \frac{q_{ult}}{F} \quad (17)$$

The foundation is then designed so that the applied bearing pressure,  $q_{app}$ , does not exceed the allowable pressure,  $q_{all}$ , i.e.,

$$q_{app} \leq q_{all} \quad (18)$$

The value of the factor of safety depends on:

- soil type;
- site investigation;
- soil variability; and
- importance of the structure and consequences of a failure.

### **Limit State Design (LSD)**

The limit states design has begun to gain popularity among geotechnical engineers.

In the limit states design the bearing capacity of the footing is considered as part of the Ultimate Limit State (ULS). The safety of the foundation is satisfied in the ULS design by using partial safety factors for the load and strength parameters.

This is done as follows:

1. Factored loads are calculated by multiplying specified loads and forces by load factors obtained from the respective codes (e.g. NBCC or OHBDC). The factored applied pressure is obtained by dividing the factored load by the base area of the footing.
2. Design shear strength parameters are calculated by multiplying the shear strength parameters of the soil by resistance factors. The design capacity of the foundation is calculated using the design shear strength parameters.
3. To satisfy the ULS, the design capacity must be equal to or greater than the factored applied pressure.

The load factors are 1.25 for Dead Load (DL), 1.5 for Live Load (LL), Wind Load (WL) and Seismic Load (SL). The resistance factors are 0.8 for angle of internal friction ( $f_\phi$ ) and 0.5 to 0.7 for cohesion ( $f_c$ ).